2020-1067 IJOI
http://www.ijoi-online.org/

International Journal of
Organizational Innovation

USING FUZZY TIME-SERIES PREDICTION FOR FORECASTING RAINFALL IN TAIWAN

I-Jyh Wen
Department of Civil and Construction Engineering, National Yunlin University of Science & Technology, No. 123, Sec. 3, University Road, Douliu City, Yunlin County 640, Taiwan (R.O.C.)

Chin-Hsing Yang*
Graduate School of Engineering Science and Technology, National Yunlin University of Science & Technology, No. 123, Sec. 3, University Road, Douliu City, Yunlin County 640, Taiwan (R.O.C.)
*Corresponding author: E-mail addresses: wenij@yuntech.edu.tw (I.J. Wen), cwn.yang@msa.hinet.net (C.H. Yang).

Abstract

Forecasting is an important academic research topic and method. This method is widely applied in various research fields, such as research fields related to industry, commerce, agriculture, economics, medicine, the environment, social science, and engineering. Scholars have proposed different analytical methods for conducting forecasting research. The proposed methods include quantitative methods, such as questionnaire or field surveys, which provide statistical data, and qualitative methods, such as interviews or observations, which provide analytical data. However, considerable labor, material resources, and time are required to acquire the necessary quantitative statistical data or qualitative analytical data for conducting forecasting research. In the era of globalization and rapidly changing virtual networks, forecast research is crucial. Therefore, rapidly obtaining research data and applying soft computing and effective prediction tools are essential. In this study, the average annual rainfall in Taiwan for 15 years (2005–2019), which is recorded in the statistical database of the Taiwanese government's official website, was used as an example for forecasting. Soft computing involving fuzzy time-series prediction was used to examine empirically the applicability of a fuzzy time-series prediction model for the prediction of rainfall data. Only 15 historical time-series data points were used to perform the prediction. The use of 10–14 fuzzy membership subsets yielded a prediction error of less than 8%, which indicated that a small amount of data can be used to obtain favorable forecasting results with the adopted method.

Keywords: Fuzzy time series, Rainfall forecast, Disaster prevention, Construction period

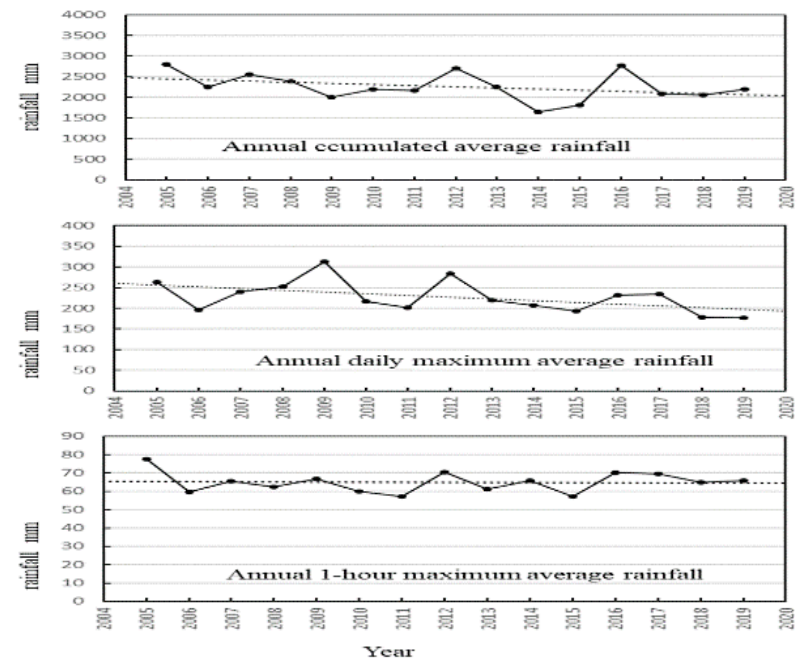
Introduction

Due to the intensified effects of extreme climate change worldwide in recent years, the loss of human life and property is increasing each year. Even countries with advanced technologies, such as the United States, the United Kingdom, France, and Germany, are unable to effectively prevent the natural disasters caused by extreme climate. Single-day or short-term heavy rainfall and the massive rainfall caused by hurricanes or typhoons cause massive losses in life and property around the world. Non heavy rainfall may affect or delay the construction schedule of projects, such as civil engineering, construction, and water conservancy projects. Reduced or insufficient rainfall also affects human livelihood, agricultural irrigation, and industries. Accordingly, if a simple forecast analysis model can effectively predict the annual rainfall, maximum rainfall per day, and 1-hour maximum rainfall, suitable response measures can be developed for relevant disaster prevention. Moreover, disaster relief units and related industries, such as the construction industry, can prepare for floods or insufficient rainfall in the following year and thus reduce the losses caused by floods or droughts.

2. Research method and material

The climate monitoring report released by the Central Weather Bureau Ministry of Transportation and Communications (2019), Taiwan, in December 2019 was adopted in this study. The relevant time-series tendency charts were plotted according to the measured time-series average of the annual accumulated rainfall, maximum daily rainfall, and 1-hour maximum rainfall recorded by 13 flatland stations in Taiwan (2005–2019) (Figure 1) (2020, March 18). In addition, the rainfall classification and precautions in Taiwan (Table 1) was the basis for judging and assessing whether rainfall would cause disasters. The relevant literature on fuzzy time-series prediction models was examined to develop a model for calculating the predicted level of the annual cumulative average rainfall, maximum annual daily (24 hours) rainfall, and annual 1-hour maximum average rainfall in Taiwan. Furthermore, the relevant forecast values for the following year were predicted. The main research objectives of this were as follows:

1.To establish a model that can effectively predict the annual rainfall with the fuzzy time-series method and to use the statistical data of the measured time-series average of the annual cumulative rainfall, maximum daily rainfall, and 1-hour maximum rainfall in Taiwan for analyzing the predictions of the fuzzy time-series prediction model.
2.To provide prediction models that can serve as references and can be used by disaster prevention and relief units as well as personnel (e.g., firefighting units, military, police, industries such as construction and engineering, and relevant researchers).



Note: Represented by the average rainfall recorded by 13 flatland stations in Taiwan

Figure 1. Time-series curve of the annual average rainfall in Taiwan.

Table 1. Rainfall classification and precautions in Taiwan.

Name	Rainfall	Precautions
Heavy rain	80 mm/24 hours or 40 mm/1 hour	Mountainous or geologically vulnerable areas: flash floods, falling rocks, and landslides may occur

		and above	Flatland: poor drainage or higher tendency of waterlogging and flood in lowland Raining areas: strong gusts and lightning strikes
Extremely heavy rain		200 mm/24 hours or 100 mm/3 hours and above	Mountainous areas: flash floods, falling rocks, landslides, and mudflows Flatland: extremely high tendency of water-logging and flood Raining areas: poor visibility, strong gusts, lightning strikes, and hail
	Torrential rain	350 mm/24 hours or 200 mm/3 hours and above	Flash floods, falling rocks, landslides, mud-flows, or collapses Flatland: increased flooded area Raining areas: poor visibility, strong gusts, lightning strikes, and hail
	Extremely torrential rain	500 mm/24 hours and above	Mountainous areas: large-scale flash floods, falling rocks, landslides, mudflows, or col-lapses Flatland: flooding with aggravating situation Raining areas: extremely poor visibility, strong gusts, lightning strikes, and hail

Note: Data compiled from the website of the Central Weather Bureau, Ministry of Transportation and Communications (revised version on March 1, 2020) (2020, March 18).

Fuzzy time series

Literature review

The fuzzy theory, which was first proposed by Zadeh (1965), has been used to develop various research methods. The fuzzy theory is based on a fuzzy set, and its main principle is harmonic tolerance. By contrast, the conventional set theory emphasizes a crisp set. In the definition of a fuzzy set for an element X, the degree to which X belongs to a set is expressed as $\mu(x)$. In other words, X corresponds to the range [0,1], with a level closer to 1 indicating a greater probability of a set containing X. The aforementioned value is called the degree of membership, and $\mu(x)$ is known as the membership function. The set is a conventional explicit set when the value of the membership function (x) is only 0 and 1. Zadeh (1975) explained that for complex or difficult-to-define situations, many qualitative standards in real life lead to difficulties in using a conventional quantitative approach for providing a reasonable and clear explanation. Accordingly, such situations must be addressed using the perspective of linguistic variables.

According to Song and Chissom (1993b), the dynamic process of defining and studying linguistic values from the perspective of Zadeh (1973) is a fuzzy time series. They believed that the biggest difference between conventional and fuzzy time series is that the conventional time series comprise real numbers, whereas fuzzy time series comprise the linguistic variable of a fuzzy set. Therefore, Song and Chissom (1993a, 1994) used the fuzzy theory of Zadeh as their research foundation to define the basic structure of the fuzzy time-series model. They proposed different fuzzy time-series prediction methods for different types of time-series data. For example, Song and Chissom explained the process and method of constructing fuzzy time-series prediction models by considering the time-series data of freshman enrollment at the University of Alabama in the United States.

Chen (1996) believed that in the prediction method proposed by Song and Chissom (1993a). Defuzzification using max–min calculation is overly complicated. Accordingly, a new and relatively simple fuzzy time-series prediction model is modeled. The freshman data used by Song and Chissom (1993a, 1994) was also adopted in the aforementioned research. The empirical results indicated that Chen's research method not only yielded superior results for the prediction of the number of freshmen enrolled but also provided satisfactory prediction results when the historical data were incomplete.

Hwang, Chen, and Lee (1998) also adopted the data variance in the fuzzy time series to propose another method for researching forecasting models. They also adopted the data used by Song and Chissom (1993a, 1994) as their research data. The aforementioned authors used the increase and decrease in the number of freshmen enrolled in the previous year as the basis for predicting changes in the number of freshmen in the new academic year. The relevant fuzzy logic set was determined using the changes in the number of freshmen enrolled in the previous year. This set was then used as the criterion for predicting the changes in the number of freshmen enrolled in the following year. The mean absolute percentage error (MAPE) level of this model was then compared with those of different fuzzy time-series prediction models, as presented in Table 2. The method proposed by Hwang, Chen, and Lee was superior to those of Chen as well as

Song and Chissom and exhibited a favorable forecast level. In addition, the time complexities of different research methods were examined to determine their ease of computation. The results indicated that the method proposed by Hwang, Che, and Lee was superior to those proposed by Song and Chissom as well as Markov.

Table 2. A comparison of the average forecasting errors of different forecasting methods.

Literature and research methods	Song and Chissiom (1993a) method	Song and Chissiom (1994) method (under model basis w = 4 and using neural net method)	Chen's (1996) method	Sullivan and Woodall (1994) Markov method	Hwang et al. (1998) method (under window basis w = 4)
Style	Time-invariant	Time-variant	Time-invariant	Time-invariant	Time-variant
Time complexity	$O(kn^2)$	$O(kn^2)$	$O(p)$	$O(cn^2)$	$O(wn)$
Average forecasting errors	3.20%	4.37%	3.22%	2.60%	3.12%

Note: k denotes the number of fuzzy logical relationships, n denotes the number of elements in the universe of discourse, p denotes the number of fuzzy logical relationship groups, c denotes the number of transitions in the historical data, and w denotes the window basis. Reference from: Hwang et al. (1998).

Fuzzy time-series prediction model

k denotes the number of fuzzy logical relationships, n denotes the number of elements in the universe of discourse, p denotes the number of fuzzy logical relationship groups, c denotes the number of transitions in the historical data, and w denotes the window basis. Reference from: Hwang et al. (1998).Table 2 presents a comparison of the forecast level obtained from the research results of various fuzzy time-series prediction models. Despite the differences in the mean error percentages of the forecast, all the fuzzy time-series prediction models possessed fairly favorable forecast levels. According to the MAPE forecast level proposed by Lewis (1982) (Table 3), the fuzzy time-series prediction models displayed favorable forecast levels.

In this study, Chen's method was used. This method is simple and clear but has a satisfactory forecast level for analysis and calculation, as presented in Table 2. The forecast level of rainfall for the following year in Taiwan was obtained with the aforementioned method by using the average time- series data of relevant rainfall statistics from the 13 flatland observation stations in Taiwan. The data were obtained through the meteorological observation data query of the Central Weather Bureau Ministry of Transportation and Communications (2020).

Table 3: MAPE forecast level (Lewis, 1982)

MAPE (%)	<10	10-20	20-50	>50
Forecast level	Extremely favorable forecast	Favorable forecast	Reasonable forecast	Incorrect forecast

The analysis and prediction steps were as follows.

Step 1: Let U be the universe of discourse for a time series. The discourse is represented as follows: $U = \{u_1, u_2,..., u_n\}$, where u_n is the subset of the universe U. Let D_{max} and D_{min} be the maximum and minimum time-series values in the universe, respectively. The full range is then expressed as follows: $R = [D_{min}- D_1, D_{max} + D_2]$, where D_1 and D_2 are appropriate positive numbers such that $D_{min} - D_1$ and $D_{max} + D_2$ are integers. The group range is then made a constant for an appropriate number of groups according to the full-range value. The full range is then grouped according to the appropriate number of groups (u_1, u_2,..., u_n).

According to the December 2019 Climate Monitoring Report of the Central Weather Bureau Ministry of Transportation and Communications (2020), the average annual rainfall measured at 13 flatland stations in Taiwan from 2005 to 2019 (Table 4) were used to calculate the statistical data for 15 years to obtain the analysis data for the universe of discourse (U). In the calculation, $D_{max} = 2801$ and $D_{min} = 1643$. The appropriate positive numbers $D_1 = 43$ and $D_2 = 99$ were adopted to obtain R = [1643 – 43, 2801 + 99] = [1600, 2900]. Subsequently, according to the full range of 2900 – 1600 = 1300, the group range was set to 130. The full range was divided into 10 groups, namely u_1 = [1600, 1730], u_2 = [1730, 1860], u_3 = [1860, 1990], u_4 = [1990, 2120], u_5 = [2120, 2250], u_6 = [2250, 2380], u_7 = [2380, 2510], u_8 = [2510, 2640], u_9 = [2640, 2770], and u_{10} = [2770, 2900].

Step 2: Let A_1, A_2, …A_k be the fuzzy sets of different linguistic variables. These fuzzy sets are expressed as follows:

$$A_1 = a_{11}/u_1 + a_{12}/u_2 + a_{13}/u_3 \dots + a_{1n}/u_n$$
$$A_2 = a_{21}/u_1 + a_{22}/u_2 + a_{23}/u_3 \dots + a_{2n}/u_n$$
$$A_k = a_{k1}/u_1 + a_{k2}/u_2 + a_{k3}/u_3 \dots + a_{kn}/u_n$$

where a_{ij} represents the size of the membership function of u_j under the

fuzzy set A_i and $a_{ij} \in [0,1]$, $1 \le i \le k$, $1 \le$

$j \le n$, and $k = n$. The series values are rewritten using the fuzzy sets in accordance with the interval they fall in so that the data become a fuzzy semantic-type time series, as presented in Table 4 The fuzzy set A_k of the linguistic variable corresponding to the subset u_n of the actual statistical values of each year ($Y(t)$; t = 2005, 2006, 2007,…, 2019) is filled into the fuzzy set field. For example, Y(2010) = 2195 belongs to the subset u_5 and its corresponding linguistic variable fuzzy subset is A_5. The basic principle of prediction in this study is that the fuzzy set of the previous time-series value $Y(t-1)$ affects the time-series value $Y(t)$; thus, the fuzzy logic relationship of each year is set to $Y(t-1) \rightarrow Y(t)$ to obtain a fuzzy logic relationship $A_i \rightarrow A_k$. Consequently, the (t −1)th time series belongs to the fuzzy set A_i and the tth time series belongs to the fuzzy set A_k.

Step 3: Establish the fuzzy logical relationship groups. According to the fuzzy logic relationship $A_i \rightarrow A_k$ established by the fuzzy sets in each period, the various fuzzy logical relationships obtained in step 2 (Table 4) correspond to the grouping of fuzzy logic relationships presented in Table 5. The group number is the number of groups determined according in the calculation in step 1. If no relevant fuzzy set is observed in Table 4 after analyzing the relevant fuzzy membership relationship and if the calculated example is the third fuzzy set A_3, then Group 3 in the fuzzy logical relationship group is presented using a space or labeled as $A_3 \rightarrow$.

Step 4: Analyze and calculate the predicted value of $F(t)$ (t = 2005, 2006, 2007, …, 2019).

The analysis and calculation in this step are based on the following three principles.

(1) If the fuzzy set at the $(t-1)$th period is A_i and only one type of fuzzy logic relationship ($A_i \rightarrow A_k$) exists, the median (m_n) of the subset u_n that has a membership relationship with A_k is taken as the forecast value. For example, the fuzzy logical relationship group of Group 2 in Table 5 is $A_2 \rightarrow A_{10}$. Thus, when the measured annual accumulated average rainfall $Y(t-1)$ falls under u_2 (belonging to the fuzzy set A_2), the accumulated average rainfall in the next year is the median ($m_{10} = 2835$) of the subset u_{10} to which A_{10} belongs.

(2) If the fuzzy set at the $(t-1)$th period is A_i but it contains more than two types of fuzzy logic relationships ($A_i \rightarrow A_{k1}$, $A_i \rightarrow A_{k2}$,... $A_i \rightarrow A_{kp}$), the arithmetic mean of the medians m_{n1}, m_{n2}, …m_{np} of the subsets u_{n1}, u_{n2}, …u_{np} to which A_{k1}, A_{k2}, …A_{kp} belong, respectively, are taken as the forecast value of the tth period. For example, in Table 5, the fuzzy logical relationship group of Group 4 is one-to-many ($A_4 \rightarrow A_4$ and $A_4 \rightarrow A_5$). Therefore, when the annual accumulated average rainfall $Y(t-1)$ falls under u_4 (belonging to the fuzzy set A_4), the forecast value of the accumulated average rainfall in the next year $F(t)$ is the arithmetic mean of the medians of u_4 and u_5 to which A_4 and A_5 belong ($\frac{1}{2}(m_4 + m_5)$

$= \frac{1}{2}(2055 + 2185) = 2120$).

(3) If no fuzzy logical relationship is observed in the fuzzy set A_i in the $(t-1)$th period, the median (m_n) of the subset u_n with which A_i shares a membership relationship is taken as the forecast value. For example, in Table 5, Group 3 does not have a fuzzy logical relationship group, that is, $A_3 \rightarrow \emptyset$. When $Y(t-1)$ falls under u_3 (belonging to the fuzzy set A_3), the predicted $F(t)$ in the next year is the median ($m_3 = 1925$) of the subset u_3 to which A_3 belongs.

The forecast level used in this study was determined from the MAPE, which is calculated as follows:

$$MAPE = \frac{1}{n-1} \left(\sum_{i=2}^{n} \left|\frac{Y(t)_i - F(t)_i}{Y(t)_i}\right| \times 100\right)$$

Results

Fuzzy time-series prediction of the annual cumulative average rainfall

The average time-series values for the rainfall statistics (2005–2019) of 13 flatland stations in Taiwan, which were obtained from the meteorological observation data query of the Central Weather Bureau Ministry of Transportation and Communications (2020), were calculated using the fuzzy time-series prediction model proposed by Chen (1996). According to the analysis characteristics of the model, the more the number of fuzzy membership subsets, the smaller is the group range of the relevant membership. Under the fuzzy set domain classification of the historical time-series data, the predicted values of following year should be close to the actual values. Thus, the error percentage can be reduced following an increase in the number of fuzzy membership subsets. In addition, the number of groups in the membership subset of the adopted method is obtained according to the set value of the full range (*R*) of the historical time-series data. Table 6 presents the analysis and calculation results of this study. The forecast error percentage for the prediction of the annual accumulated average rainfall in Taiwan for different membership subsets was 11.98%, 8.03%, 6.36%, and 7.17% when $w = 5$, 8, 10, and 13, respectively. As presented in Table 3, the different numbers of groups yielded extremely favorable forecast levels, except when the number of groups was 5 (favorable forecast level). A comparison of the forecast level acquired for four numbers of groups indicates that a superior forecast level (error of 6.36%) was obtained when $w = 10$. Furthermore, the unknown annual accumulated average rainfall for the following year (2020) was predicted. For example, the annual accumulated average rainfalls were 2315, 2169, 2185, and 2450 mm when $w = 5$, 8, 10, and 13, respectively. Figure 2 displays the trends in the measured and predicted values for the different numbers of groups in different membership subsets. The lowest MAPE of 6.36% was obtained when $w = 10$. When $w = 10$, the annual accumulated average rainfall in Taiwan in 2020 (2185 mm) was predicted to be marginally lower than that in 2019 (2197 mm). The measured annual accumulated average rainfall curve in Figure 1 illustrates a declining trend, which indicates that the future supply of water resources in Taiwan may decrease. This trend can serve as a reference for relevant units or personnel.

Fuzzy time-series prediction of the annual maximum daily rainfall

The time-series data (2005–2019) of the annual daily maximum average rainfall at 13 flatland stations in Taiwan were calculated using the fuzzy time-series prediction model of Chen (1996). The model of Chen (1996) was also used for calculating the membership subset group numbers ($w = 5$, 8, 10, and 14), and the results are presented in Table 7. The forecast error percentages for the annual daily maximum average rainfall in Taiwan were

8.74%, 7.74%, 6.57%, and 4.46% when w = 5, 8, 10, and 14, respectively. According to Table 3, all the aforementioned percentages represent a favorable forecast level. Moreover, a comparison of the forecast level of the time-series data for different w values indicated that a higher number of groups yielded a more favorable forecast level. For example, w = 14 yielded a forecast level of 4.46%. Furthermore, the predicted unknown annual daily maximum average rainfall for the following year (2020) was predicted to be 189, 183.8, 182, and 180 when w = 5, 8, 10, and 14, respectively. Figure 3 displays the trends for the measured and predicted values for different

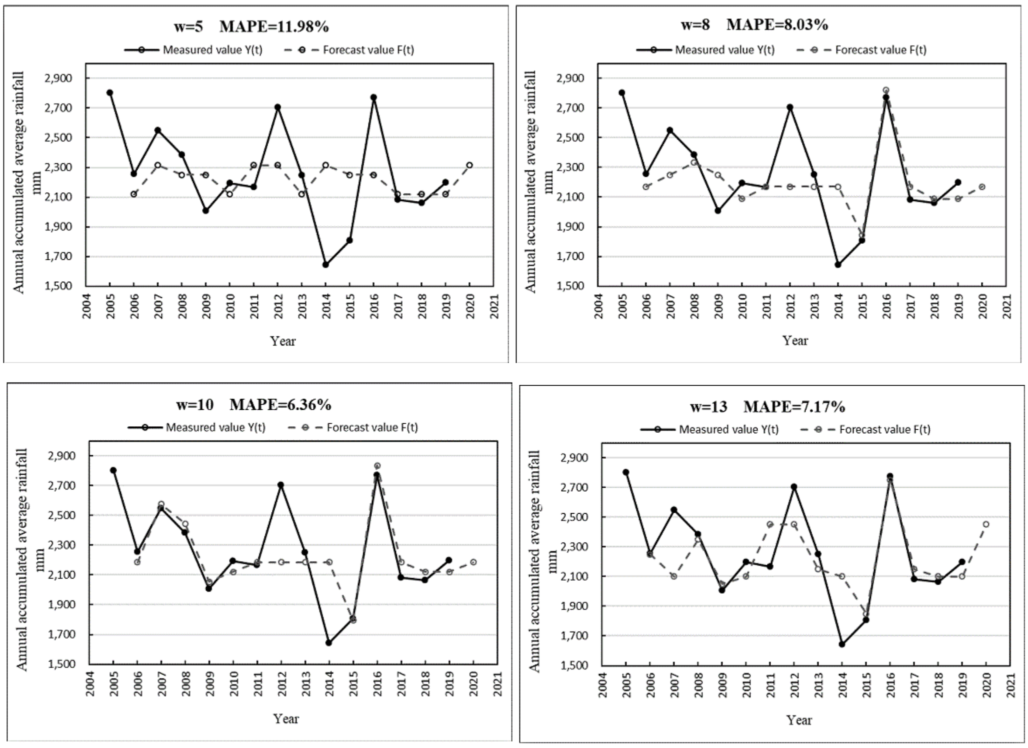


Figure 2. Curve of the forecasting result for the annual accumulated average rainfall in Taiwan.

numbers of groups. The lowest MAPE of 4.46% was obtained for w = 14. When w = 14, the predicted annual daily maximum average rainfall in Taiwan in 2020 (180 mm) was slightly higher than that in 2019 (176.6 mm). According to Table 1, an annual daily maximum average rainfall of 180 mm falls under extremely heavy rain but is also close to torrential rain. The aforementioned information can serve as a reference for the relevant disaster prevention and relief units or personnel.

Fuzzy time-series prediction of the annual maximum average rainfall in 1 hour

The fuzzy time-series prediction model of Chen (1996) was used to calculate the time-series data (2005–2019) of the annual 1-hour maximum average rainfall for 13 flatland stations in Taiwan. The model of Chen (1996) was also used to calculate the membership subset group numbers (*w* = 5, 8, 10, and 12), and the results are presented in Table 8. The forecast error percentages of the annual daily maximum average rainfall in Taiwan were 5.43%, 4.38%, 4.74%, and 3.65% when *w* = 5, 8, 10, and 12, respectively. According to Table 3, all the aforementioned percentages represented a favorable forecast level. The lowest forecast error of 3.65% was obtained when the number of groups 12. In general, the accuracy of the results improved with the number of groups. However, a *w* value of 8 provided a higher forecast level than a *w* value of 10. Furthermore, the annual 1-hour maximum rainfalls predicted for the following year (2020) were 60, 61.3, 61.3, and 59.5 mm when *w* = 5, 8, 10, and 12, respectively. Figure 4 illustrates the trends of the measured and predicted values for different numbers of groups. When *w* = 12, The annual 1-hour maximum average rainfall predicted for 2020 (59.5 mm) was lower than that predicted for 2019 (65.9 mm). According to Table 1, an annual 1-hour maximum average rainfall of 59.5 mm falls under heavy rain but is also close to torrential rain. The aforementioned result can serve as a reference for the relevant disaster prevention and relief units or personnel.

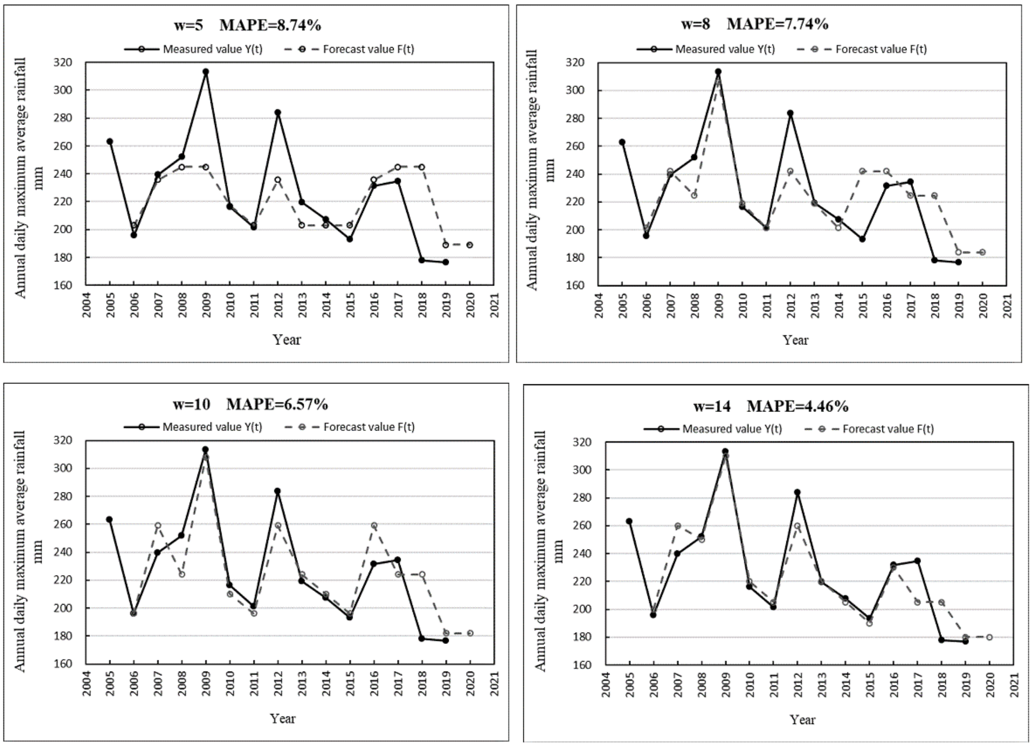


Figure 3. Curve of the forecasting results for the annual daily maximum average rainfall in Taiwan.

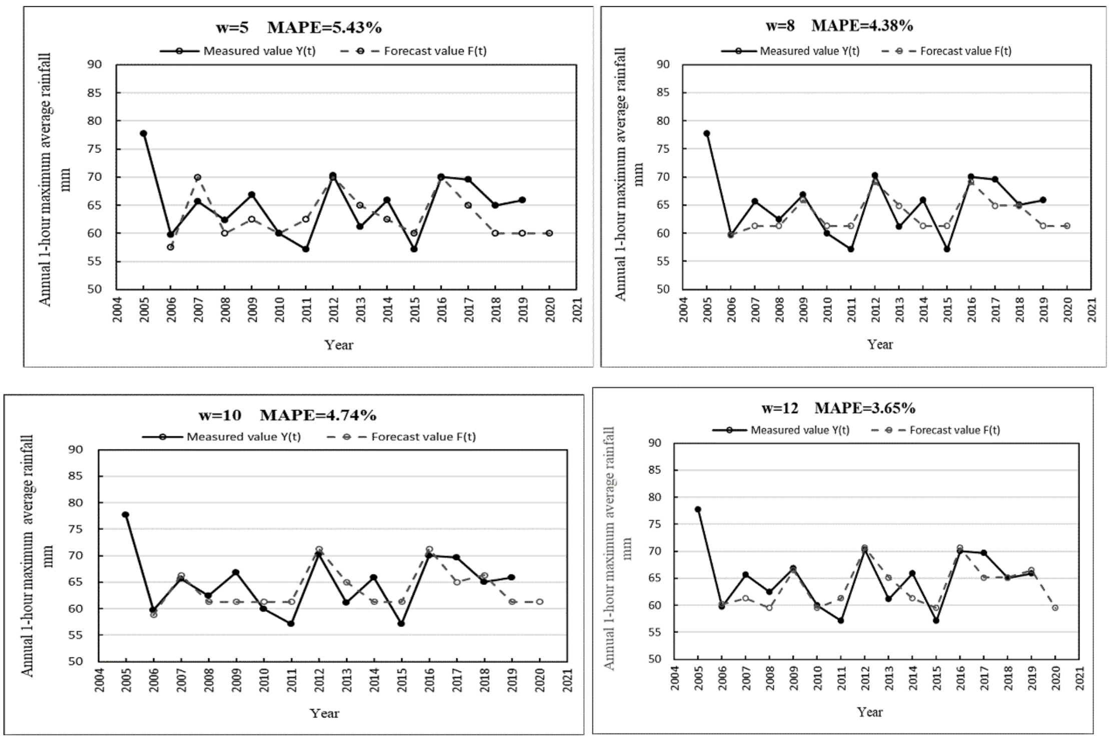


Figure 4. Curves of the forecasting results of the annual 1-hour maximum average rainfall in Taiwan.

Conclusion and Suggestions

The forecast levels of the results obtained with the fuzzy time-series prediction model of Chen (1996) when using different fuzzy membership subset group numbers for the same historical time-series data were compared. The comparison revealed that regardless of the differences between the measured and predicted values for the annual accumulated average rainfall, daily (24-hour) maximum average rainfall, or 1-hour maximum average rainfall, a higher number of groups in the fuzzy membership sets provided a more favorable forecast level and a smaller error percentage between the measured and predicted values. Some numbers of groups with optimal forecast levels did not yield favorable results due to their high value. Thus, the author suggests that before predicting the next unknown number, different numbers of groups can be first analyzed to acquire the most favorable forecast level. The relevant prediction and analysis can then be performed using the optimal number of groups.

The research results revealed that the setting of the maximum and minimum boundary values in the full range (*R*) affected the forecast level. For example, when the predicted annual daily maximum average rainfall was *R* = [175, 315], the forecast levels obtained were 8.74%, 7.74%, 6.57%, and 4.46%, when *w* = 5, 8, 10, and 14 respectively. However, if *R* = [170, 320], the forecast level were 13.45%, 10.39%, 7.26%, and 5.36% when *w* = 5, 8, 10, and 14, respectively. When the predicted annual 1-hour maximum average rainfall was *R* = [55, 80], the forecast levels were 5.43%, 4.38%, 4.74%, and 3.65% when *w* = 5, 8, 10, and 12, respectively. However, when *R* = [50, 80], the forecast levels for each group were 5.44%, 4.25%, 4.33%, and 4.74% when *w* = 5, 8, 10, and 12, respectively. The aforementioned results indicated that when selecting the maximum and minimum boundary values of the full range (*R*) for the forecast in this study, the forecast level of different full range values should be analyzed first to determine the most favorable forecast level. The relevant forecast and analysis can then be performed using the optimal *R* values. Overall, the forecast levels obtained using the prediction model of Chen (1996) when *w* = 10, 12, 13, and 14 had errors of less than 8%; thus, the forecast levels were favorable.

The conclusions of this study are as follows:

1. The annual average rainfall in Taiwan was forecasted with the fuzzy time-series prediction model by using the data of 15 historical time se-

ries and 10–14 groups of fuzzy membership subsets. The error percentage obtained with the aforementioned model was higher than those of the prediction models in Table 2. However, the error was still less than 8%, which indicated that the aforementioned model is a favorable forecast model.

2. The annual average rainfall in Taiwan was analyzed and predicted in this study. The annual accumulated average rainfall exhibited a declining trend each year at the 13 flatland stations. Moreover, although the predicted annual daily maximum average rainfall and 1-hour maximum rainfall in the following year fell under the classification of heavy rain, these rain-fall values were also close to the level of torrential rain. This finding can serve as a reference for relevant disaster prevention and relief units or personnel to implement appropriate response measures.

3. In addition, this discovery can also provide a reference for the actual construction period assessment of construction projects and the implementation of appropriate response measures for disaster prevention and relief.

1. References

Central Weather Bureau Ministry of Transportation and Communications. (2019). 2019 Taiwan Climate Analysis, *Monthly Report on Climate System*, *130*, 12–16.

Central Weather Bureau Ministry of Transportation and Communications. (2020, March 18). https://www.cwb.gov.tw/V8/C/C/Statistics/monthlydata.html.

Chen, S. M. (1996). Forecasting enrollments based on fuzzy time series. *Fuzzy Sets and System*, *81*, 311–319.

Hwang, J. R., Chen, S. M., & Lee, C. H. (1998). Handling forecasting problems using fuzzy time series. *Fuzzy Sets and System*, *100*, 217–228.

Lewis, C. D. (1982). *Industrial and business forecasting methods: A practical guide to exponential smoothing and curve fitting*. London: Butterworth-Heinemann.

Song, Q., & Chissom, B. S. (1993a). Forecasting enrollments with fuzzy time series – part I. *Fuzzy Sets and System*, *54*, 1–9.

Song, Q., & Chissom, B. S. (1993b). Fuzzy time series and its models. *Fuzzy Sets and System*, *54*, 269–277.

Song, Q., & Chissom, B. S. (1994). Forecasting enrollments with fuzzy time series – part II. *Fuzzy Sets and System*, *62*, 1–8.

Sullivan, J., & Woodall, W. H. (1994). A comparison of fuzzy forecasting and Markov modeling. *Fuzzy Sets and System*, *64*, 279–293.

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, *8*, 338–353.

Zadeh, L. A. (1973). Outline of a new approach to the analysis of complex systems and decision processes. *IEEE Transactions on Systems, Man and Cybernet*, *3*, 28–44.

Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning. *Information Sciences*, *8*, 199–249.

Table 4. Measured annual accumulated average rainfall in Taiwan (2005–2019) and the relevant fuzzy sets.

Year t	Cumulative measured average rainfall *Y(t)* (mm)	Membership subset u_n	Median m_n	Median A_k	Fuzzy logic relationship $A_i \rightarrow A_k$
2005	2,801	u_{10} = [2770, 2900]	2,835	A_{10}	
2006	2,255	u_6 = [2250, 2380]	2,315	A_6	$A_{10} \rightarrow A_6$
2007	2,548	u_8 = [2510, 2640]	2,575	A_8	$A_6 \rightarrow A_8$
2008	2,385	u_7 = [2380, 2510]	2,445	A_7	$A_8 \rightarrow A_7$
2009	2,005	u_4 = [1990, 2120]	2,055	A_4	$A_7 \rightarrow A_4$
2010	2,195	u_5 = [2120, 2250]	2,185	A_5	$A_4 \rightarrow A_5$
2011	2,165	u_5 = [2120, 2250]	2,185	A_5	$A_5 \rightarrow A_5$
2012	2,705	u_9 = [2640, 2770]	2,705	A_9	$A_5 \rightarrow A_9$
2013	2,249	u_5 = [2120, 2250]	2,185	A_5	$A_9 \rightarrow A_5$
2014	1,643	u_1 = [1600, 1730]	1,665	A_1	$A_5 \rightarrow A_1$
2015	1,808	u_2 = [1730, 1860]	1,795	A_2	$A_1 \rightarrow A_2$
2016	2,772	u_{10} = [2770, 2900]	2,835	A_{10}	$A_2 \rightarrow A_{10}$
2017	2,083	u_4 = [1990, 2120]	2,055	A_4	$A_{10} \rightarrow A_4$
2018	2,062	u_4 = [1990, 2120]	2,055	A_4	$A_4 \rightarrow A_4$
2019	2,197	u_5 = [2120, 2250]	2,185	A_5	$A_4 \rightarrow A_5$

Table 5. Fuzzy logical relationship groups.

Group1:	$A_1 \rightarrow A_2$		
Group2:	$A_2 \rightarrow A_{10}$		
Group3:	$A_3 \rightarrow$		
Group4:	$A_4 \rightarrow A_4$	$A_4 \rightarrow A_5$	
Group5:	$A_5 \rightarrow A_1$	$A_5 \rightarrow A_5$	$A_5 \rightarrow A_9$
Group6	$A_6 \rightarrow A_8$		
Group7	$A_7 \rightarrow A_4$		
Group8	$A_8 \rightarrow A_7$		
Group9	$A_9 \rightarrow A_5$		
Group10	$A_{10} \rightarrow A_4$	$A_{10} \rightarrow A_6$	

Table 6. Forecast results for the annual accumulated average rainfall in Taiwan.

Year (t)	Measured value Y(t)	w = 5	w = 8	w = 10	w = 13
		Forecast value F(t)	Forecast value F(t)	Forecast value F(t)	Forecast value F(t)
2005	2,801				
2006	2,255	2,120	2,169	2,185	2,250
2007	2,548	2,315	2,250	2,575	2,100
2008	2,385	2,250	2,331	2,445	2,350
2009	2,005	2,250	2,250	2,055	2,050
2010	2,195	2,120	2,088	2,120	2,100
2011	2,165	2,315	2,169	2,185	2,450
2012	2,705	2,315	2,169	2,185	2,450
2013	2,249	2,120	2,169	2,185	2,150
2014	1,643	2,315	2,169	2,185	2,100
2015	1,808	2,250	1,844	1,795	1,850
2016	2,772	2,250	2,819	2,835	2,750
2017	2,083	2,120	2,169	2,185	2,150
2018	2,062	2,120	2,088	2,120	2,100
2019	2,197	2,120	2,088	2,120	2,100
2020	Unknown value	2,315	2,169	2,185	2,450
	MAPE	11.98%	8.03%	6.36%	7.17%

Note: Full range (R) of the data in this group = $[D_{min} - D_1, D_{max} + D_2]$ = [1643 – 43, 2801 + 99] = [1600, 2900] = 1300. In the table, w represents the number of groups of the membership subset u_n used when performing prediction in the fuzzy time series. For example, w = 10 represents 10 groups. MAPE(Lewis, 1982) is the mean absolute percentage error between the measured and predicted values for the data of 2005–2019.

Table 7. Forecasting results for the annual daily maximum average rainfall in Taiwan.

Year (t)	Measured value Y(t)	w = 5 Forecast value F(t)	w = 8 Forecast value F(t)	w = 10 Forecast value F(t)	w = 14 Forecast value F(t)
2005	263.0				
2006	195.8	203.0	201.3	196.0	200.0
2007	239.6	235.7	242.1	259.0	260.0
2008	252.0	245.0	224.6	224.0	250.0
2009	313.3	245.0	306.3	308.0	310.0
2010	216.3	217.0	218.8	210.0	220.0
2011	201.4	203.0	201.3	196.0	205.0
2012	283.8	235.7	242.1	259.0	260.0
2013	219.4	203.0	218.8	224.0	220.0
2014	207.4	203.0	201.3	210.0	205.0
2015	193.2	203.0	242.1	196.0	190.0
2016	231.5	235.7	242.1	259.0	230.0
2017	234.5	245.0	224.6	224.0	205.0
2018	177.9	245.0	224.6	224.0	205.0
2019	176.6	189.0	183.8	182.0	180.0
2020	Unknown value	189.0	183.8	182.0	180.0
	MAPE	8.74%	7.74%	6.57%	4.46%

Note: Full range (R) of the data in this group = [$D_{min} - D_1$, $D_{max} + D_2$] = [176.6 – 1.6, 313.3 + 1.7] = [175, 315] = 140. In the table, w represents the number of groups of the membership subset u_n used when performing prediction in the fuzzy time series. For example, w = 10 represents 10 groups. MAPE(Lewis, 1982) is the mean absolute percentage error between the measured and predicted values for the data of 2005–2019.

Table 8. Forecasting results of the annual 1-hour maximum average rainfall in Taiwan.

Year (t)	Measured value Y(t)	w = 5	w = 8	w = 10	w = 12
		Forecast value F(t)	Forecast value F(t)	Forecast value F(t)	Forecast value F(t)
2005	77.7				
2006	59.7	57.5	59.7	58.8	60.2
2007	65.6	70.0	61.3	66.3	61.3
2008	62.4	60.0	61.3	61.3	59.5
2009	66.8	62.5	65.9	61.3	66.5
2010	60.0	60.0	61.3	61.3	59.5
2011	57.1	62.5	61.3	61.3	61.3
2012	70.3	70.0	69.1	71.3	70.6
2013	61.1	65.0	64.9	65.0	65.1
2014	65.9	62.5	61.3	61.3	61.3
2015	57.1	60.0	61.3	61.3	59.5
2016	70.0	70.0	69.1	71.3	70.6
2017	69.6	65.0	64.9	65.0	65.1
2018	65.0	60.0	64.9	66.3	65.1
2019	65.9	60.0	61.3	61.3	66.5
2020	Unknown value	60.0	61.3	61.3	59.5
	MAPE	5.43%	4.38%	4.74%	3.65%

Note: Full range of the data in this group $R = [D_{min} - D_1, D_{max} + D_2]$ =[57.1 − 2.1, 77.7 + 2.3] = [55, 80] = 25. In the table, w represents the number of groups of the membership subset u_n used when performing prediction in the fuzzy time series. For example, w = 10 represents 10 groups. MAPE(Lewis, 1982) is the mean absolute percentage error between the measured and predicted values for the data of 2005–2019.